MEASURING SOCIOPOLITICAL INEQUALITY

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Equality or its absence has long been a focal concept in political philosophy. Aristotle spoke against the democratic conception of justice defined as "the enjoyment of arithmetical equality," preferring the enjoyment of proportional equality on the basis of merit or property; in effect he took a conservative, inegalitarian view. In the spirit of the French Revolution and its radical emphasis on "Liberty, Equality, and Fraternity," Thomas Paine argued that "inequality of rights has been the cause of all the disturbances, insurrections, and civil wars, that ever happened . . . ."

More contemporary discussions often implicitly or explicitly rely on definitions of equality. The U.S. Constitution calls for "equal protection" under the law. Citizens of various persuasions argue for or against racial imbalance in their schools, for or against more progressive tax structures. When sufficiently general questions are asked, the great majority of the American people is for "equality of economic opportunity" and against government by or for a privileged few. In an historic 1964 decision against legislative malapportionment (*Reynolds versus Sims,* 1964), Chief Justice Warren enunciated "the fundamental principle of representation for equal numbers of people, without regard to race, sex, economic status or place of residence within the state."[1]

THE CONTRIBUTION OF DESCRIPTIVE STATISTICS

Of course, statistics cannot in and of itself resolve the moral and political issues raised in these quotations, nor, without the explanatory theories and data of social science, can it answer the many rival claims about the causes and consequences of various kinds of inequality. But I would like to argue that as a set of ideas and practices, statistics has contributed to the clarification of the meanings of sociopolitical *inequality.* In a sense, this achievement amounts to providing a common notion of sociopolitical inequality, several measures or quantitative descriptions of the extent of inequality in a particular situation, and several criteria for choosing one measure over another as more appropriate to a particular situation or concept. After a brief review of some conventional aspects of most statistical measures of inequality, we shall present some of these measures and several criteria for choice among them.

Can we think of such modest statistical accomplishments as in any way a political success? Plato would have considered the formal ideal of justice of ultimate significance, its realization, even imperfectly in the world of politics, a noble human achievement. But is greater sociopolitical equality always greater justice? For Tom Paine it often was, for Aristotle it clearly was not. Evaluations of contemporary progress toward greater political equality or toward multiracial equality of educational opportunity are questions on which citizens disagree. Many would see such achievements as realizations of ideals of equality whose time has come, others as victories for hollow formalisms. Statistical assessments of two such controversial achievements will conclude this paper.

STATISTICAL CONVENTIONS IN MEASURING INEQUALITY

The simplest but perhaps the most important choice that statistical treatments of inequality have conventionally made is to focus on the distribution of valued objects, events, and relationships. These have included income, land, votes, legal treatment, ownership shares, and racially equivalent classmates. The abstractness of the concept of value distribution means that all these different values can be discussed in the same statistical terms.

Does this focus on value distributions seem an obvious or inevitable choice? To some, familiar with notions of income inequality, the answer may be yes. But some deeper reflection suggests that more subjective, more qualitative, and less comparable aspects of inequality might have been focused on. What did it mean, for example, to many of the suburban and black voters who were underrepresented in their state legislatures before the major reapportionment decisions of the Supreme Court? Obviously it meant more to some than to others. Except for the common facts of underrepresentation, these meanings were different for many citizens and would be hard to summarize. The same point could be made about income differences, of course. For some, an extra $5000 income would mean being able to pay urgent medical bills; for others, being able to pay for part of their son's college education, a new car, or a pleasure trip to the Taj Mahal.

[1] This and subsequent citations of the Supreme Court decisions are from *The New York Times,* June 16, 1964, pp. 28–31.

Nonetheless, the statistical orientation has focused on quantitative common denominators of meaning—a vote is a vote, a dollar a dollar. This interpretation has increased the possibility of comparisons across individuals, families, cities, states, even if it has lost significant subjective differences. Equality before the law or equality of opportunity means something concrete and comparable: similar treatment in similar circumstances.

Another aspect of most serious statistical measures of inequality is that they are, or they rely on, cumulative measures. Thus, in the interests of comparison, a statistician typically looks at the cumulative effects of inequalities in value distribution over an entire population or measures one person's privileged position in terms of how far it is above the average calculated for the whole population. Comparing the largest and the smallest incomes, for example, might make good newspaper copy, but it would not reflect the cumulative impact of the inequalities involved.

In part because of their reliance on cumulative terms, most descriptive statistical measures of inequality are calculated in generally comparable units. Thus a statistician would want to be able to compare income distributions in rubles with those in francs or dollars; to do so he relies on his knowledge of total incomes in each society. Of the several ways of getting such measures, two cumulative procedures will be used frequently here: working with data in percentage and percentile form, and defining indexes on a 0-to-1 or 0-to-100 scale by dividing the raw numerical value in particular units by its maximum value calculated in these same units. The resulting measures are pure numbers; certainly this property maximizes the possibility of comparison across variables.

SEVERAL MEASURES OF SOCIOPOLITICAL INEQUALITY

In a condensed, but geometrically intuitive, way, Figure 1 suggests at least six different but related cumulative measures of sociopolitical inequality. The data there represent the distribution of 150 seats in the New York State Assembly over a total state population of 17 million around 1960. In drawing Figure 1(a), the various Assembly districts (which have one representative each) were ordered from the largest to the smallest in population size. Because each district, regardless of population, elects one assemblyman, it follows

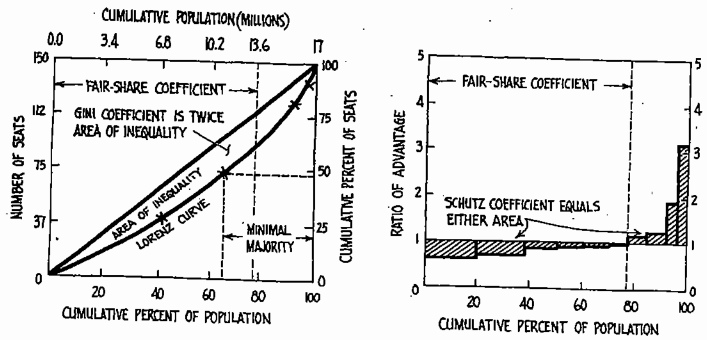

FIGURE 1

Legislative malapportionment in the New York State Assembly, 1960. Source: Alker (1965)

that the most populous districts are the most underrepresented. Plotting the cumulative percentage of seats controlled by the most underrepresented 10, 20, 30, 40, . . . , 100% of the population and then drawing a smooth curve through these points gives us our first cumulative measure of inequality: the *Lorenz curve.* Let us draw a vertical line from 40% on the horizontal axis to the curve, and from there a horizontal line to the right-hand vertical axis. The reading there shows us that the most underrepresented 40% of the population of New York in 1960 controlled only about 28% of the Assembly seats. Note that if representation were equally distributed the Lorenz curve would take the form of a straight line from the lower left corner to the upper right one, a 45° line.

The Lorenz curve also gives us a second measure of inequality: the percentage of the total held by the most "overrepresented" n% of the population (where n may be 1, 5, 8, 10, etc.). Thus Figure 1(a) shows that the most "overrepresented" 8% of the population elected 20% of the State Assemblymen! Going slightly further down the Lorenz curve gives us evidence that 35% of the state's voting population were potentially, through their representatives, a minimal majority in the Assembly.

Leaving Figure 1(a) for a moment, we realize that it has become convenient to talk about "over-" and "under-" represented citizens. A remarkable property of the Lorenz cumulative curve is that its slope (inclination, or rate of increase) provides us with a measure of this "over-" or "under-" representation. Figure 1(b), a "slope curve," is in fact a plot of the slope of the Lorenz curve shown in (a). To construct the slope curve the data are grouped into ten 15-seat categories. The "ratios of advantage" plotted vertically in (b) show, for example, that the most overrepresented 3.1% of

the population, with 15 Assemblymen, elected more than three times as many representatives as the same number of "average" citizens. For this group the value of the slope—their ratio of advantage in the bar graph (b)—is about 3.2. (A little reflection suggests that this is derivable as follows: if 3.1% of the population elects 15 Assemblymen when an "average" 10% should do so in an Assembly of 150 members, their ratio of advantage is $10/3.1 = 3.2$.)

Another quantitative measure of inequality, comparable across states, is immediately suggested by the slope curve. The *equal-share,* or *fair-share, coefficient* measures what percentage of the population gets less than its equal or fair share of representatives. In New York in 1960, it was 77% for the State Assembly. In other words, 77% of the New York citizen population was at least somewhat underrepresented!

Finally, we come to two summary measures most preferred by statistically minded social scientists, the *Gini coefficient* and the *Schutz coefficient.* The Gini coefficient is the area of inequality [between the ideal 45° line and the actual Lorenz curve in Figure 1(a)] divided by the maximum possible area. This area of inequality ranges from 0 in the case of perfect equality (the poorest 50% of the population still get 50% of the values) to ½ (when an infinitesimally small fraction of the population possesses all the values and the resulting area is the right triangle below the 45° line with sides of 100 and 100). Thus the Gini coefficient itself ranges between 0 and $½/½ = 1$. For the data in Figure 1, a Gini coefficient of 0.22 indicates that malapportionment was 22% of its theoretical maximum value.

Another geometrically appealing measure comes from Figure 1(b). Here we sum the area of advantage above the fair-share coefficient or the area of disadvantage below it. The Schutz coefficient is equal to either the total area of advantage or the total area of disadvantage. It is found by summing the areas of the rectangles to the left of the fair-share coefficient or alternatively by summing the areas of the rectangles to the right of the fair-share coefficient. For the Schutz coefficient, the minimum is obviously zero; its maximum is 1.0 (or 100.0 in percentage units). The New York data give a value of 0.156, or 15.6% for the Schutz coefficient.

All of the measures we have mentioned are in some sense aspects of cumulative quantitative value distributions. Typically they measure deviations from an ideal of perfect equality: a 45° Lorenz curve or a constant ratio of advantage equal to 1.0. But Aristotle and others would not accept the notion of perfect numerical equality. For cases such as income distribution, malapportionment, or more controversially, racial imbalance, they might argue that an appropriate ideal was less than complete equality. We might consider as inequitably or unjustly treated only those taxpayers with incomes below one-third of the national average, only those citizens with ratios of voting advantage below 0.90, or only those students with 25% fewer white classmates than the city average. In such cases, we could draw Lorenz curves showing maximum allowable inequality as a standard rather than the traditional 45° line. Slope curves could be derived from such curves, and departures beyond even these lines would then be the basis for a whole class of new cumulative measures of unacceptable inequality.

CRITERIA FOR CHOOSING AMONG MEASURES

Whether defined in egalitarian or inegalitarian terms, two important criteria argue in favor of some version of either the Gini coefficient or the Schutz coefficient. First of all, each is a full information measure in that all the data are used in calculating them, not just the top 8, 10, 20, or 50%. Second, if we assume that each datum is as significant as any other, we want measures that are sensitive to all sorts of differences among them. The minimal majority and fair-share coefficients depend wholly on the location of one point on the Lorenz curve or its slope, but the Gini and Schutz coefficients depend upon location and slope all along the Lorenz curve. More sensational measures such as "the top 1%, who have such and such . . . " can be misleading as to where the rest of the population stands.

A third criterion for choosing among measures, one offered by Yntema for the study of income inequalities, is the stability of a measure's results when different ways of grouping the data are employed. Although this may seem a technical criterion of little substantive import, for those often accustomed to getting incomplete data grouped by deciles or quartiles this criterion is an important one in practice.

Another question raised by social statisticians is whether various measures are highly intercorrelated or not. The implicit criterion is that one needs as many measures of inequality as there are philosophically important and empirically distinct aspects of the phenomenon at hand. It is of particular interest in the malapportionment case, for example, that both the Gini and the Schutz coefficients give very similar, or highly interrelated, results, with the minimal majority measure not far behind. That the fair-share coefficient is much less closely related to any of these coefficients argues for the necessity of thinking in terms of additional important and meaningful aspects of any unequal value distribution.

SOME APPLICATIONS OF STATISTICAL MEASURES

Let us briefly discuss two areas of statistical applications, malapportionment and racial imbalance, and close with some speculations about our initial quotation from Thomas Paine. Although we shall only sketch some of the more interesting findings and the extent to which controversial political achievements have grown from them, it may suffice to convey to the reader the utility of the above conventions, criteria, and measures.

First, a word is in order about New York malapportionment as it was treated by the U.S. Supreme Court. This is perhaps the clearest case of the motivating force of the quantitative equalitarian ideal—the Lorenz line of full equality—stated in "one man, one vote" terms. The close correspondence of the popular minimal majority measures with the more technically adequate Gini and Schutz coefficients in reflecting an underlying reality was seen by the majority of judges in their historic judgment that there was a violation of the Constitutional call for "equal protection" and "the right to vote." Although explicit use of such sophisticated measures is not indicated in the Court's opinion, it is clear that their opinions on *Reynolds versus Sims* reflect statistical perspectives. Even in dissent, Justice Stewart argues that "nobody has been deprived of the right to have his vote counted," while the Warren majority opinion states in quantitative language that "diluting the weight of votes because of place of residence impairs basic right under the 14th Amendment." In language remarkably like the measures we have reviewed, the majority objected to "minority control of state legislative bodies," did not believe that one person ought to "be given twice or ten times the voting power of another," and sympathized when 56% of the citizen population elected only 48% of the Assembly. Realizing that New York in 1960 was less malapportioned than most other American states in terms of the Gini, Schutz, or minimal majority coefficients, we gain some idea of the reenfranchisement that has taken place since then because of the Court's decision. This effort has not been undisputed, however; almost a majority of state legislatures have called for a Constitutional Convention on the reapportionment issue.

Table 1 and the derivative Lorenz curve in Figure 2 show how the Lorenz curve idea can be extended from the cases of income inequality and malapportionment to measure racial imbalance. The cumulative "value" represented in Figure 2 is the percentage of white students available as schoolmates; an analogous figure could have been drawn with percentages of nonwhites. Note that in this case the complete-imbalance picture we described earlier would be impossible unless there were only one white student in all of New Haven. So in order to evaluate the amount of imbalance we have, we must compare the actual Lorenz curve with the one we would have if there were complete imbalance given the existing number of white students. If we were not constrained at all by the sizes of the existing schools, then we could visualize complete imbalance as all of the black students going to all-black schools and all of the white students going to all-white schools. If we then ordered the schools by percentage of white students, we would have a Lorenz curve showing a maximum feasible imbalance that was at first completely flat along the horizontal axis and then slanted up directly to the upper right-hand corner. Because there were four junior high schools in New Haven in 1964 and because these schools had fixed capacities, it is not possible to visualize all the schools as completely segregated; we must have at least one integrated

TABLE 1. The Actual Racial Breakdown of Students in New Haven's Four Junior High Schools, June 1964*

SCHOOLS	NUMBER OF STUDENTS			PERCENTAGES				
	Whites	Non-whites	Total	Percent of All Whites	Percent of All Students	Cumulated Percent of All Whites	Cumulated Percent of All Students	Ratio: Percent Whites/ Percent Students
Bassett	55	555	610	3	17	3	17	0.18
Troup	419	514	933	19	27	22	44	0.70
Sheridan	741	148	889	34	25	56	69	1.36
Fair Haven	968	140	1108	44	31	100	100	1.42
Total	2183	1357	3540	100	100			1.0 (weighted average)

* These data are taken from a document issued by the New Haven Public Schools, Dr. Laurence G. Paquin, Superintendent, June 8, 1964, entitled *Proposals for Promoting Equality of Educational Opportunity and Dealing with the Problems of Racial Imbalance*, pp. 10–11.

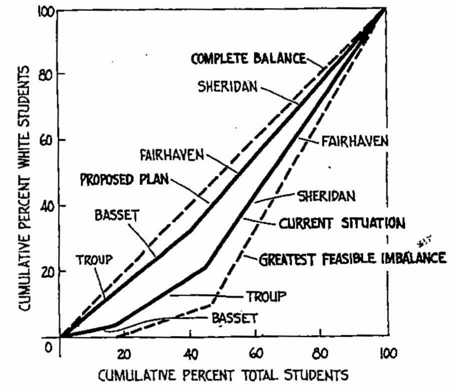

FIGURE 2
Four alternative patterns of racial imbalance in New Haven junior high schools. Source: Alker (1965)

school. Thus the curve of greatest possible imbalance is the lowest one in Figure 2. The actual racial imbalance in the system was measured by a Gini coefficient of 0.25; the maximum possible was 0.40.

Was the cumulative inequality evident in Table 1 and summarized graphically in Figure 2 sufficiently motivating to inspire desirable political action? Of what sort? The New Haven Superintendent proposed, through rezoning and selective busing, to improve racial balance to the considerable extent indicated in Figure 2. The proposed plan would reduce the Gini measure of imbalance from 0.25 to 0.09. After considerable controversy, a somewhat revised plan was evidently put into effect. Many Americans, excluding the majority of the Supreme Court, do not find racial imbalance a compelling issue, so the extent to which such action was a "success" is perhaps more controversial than state legislative reapportionment.

Finally, let us consider some inequalities that have inspired little or no "successful" political action. Recall that Thomas Paine claimed that inequality of rights "has been the cause of all civil insurrections." Statistical analyses do show land and income inequalities within nations to contribute to domestic group violence.

Whether the more extreme and growing inequality between the rich and poor nations of the world has or will occasion similar violence or corrective action remains an open question. Do the members of the poorer, mostly non-white, relatively powerless majority of the world's population have a right to a better life or to more control over their destiny? The answer lies in the success or failure of a revolutionary statistical ideal.

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